# RECOVERING THE INTERNAL DYNAMICS AND THE SHAPES OF GALAXY CLUSTERS: VIRGO CLUSTER

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We describe a method for recovering of the substructure, internal dynamics and geometrical shapes of clusters of galaxies. Applying the method to the Virgo cluster, we first, reveal the substructure of the central 4 arc degree field of the Virgo cluster by means of S-tree technique. The existence of three main subgroups of galaxies is revealed and their dynamical characteristics are estimated. Then, using the previously suggested technique (Ref.[1]), the bulk flow velocities of the subgroups are evaluated based on the distribution of the redshifts of the galaxies. The results enable us also to obtain a secure indication of the elongation of the Virgo cluster and its positional inclination.

## 1 Introduction

The present paper aims to find out a method enabling the study of the subgroupings and their bulk flows, as well as the shapes of the clusters of galaxies, based on rather general assumptions. The existence of subgroups of clusters in general is a known fact, revealed by different methods for various clusters (see e.g.[2]). The study of a sample of Abell clusters from ESO Key program (ENACS) performed by S-tree method enabled to conclude the existence of subgroups - *galaxy associations*, dynamical entities with remarkable properties as a possible common feature of clusters of galaxies[3],[4],[5].

We use the method to analyze the substructure and internal dynamics of the central region of Virgo cluster. Though the Virgo cluster because of its

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close location is one of the best studied clusters, the precise measurements of distance have been performed for relatively few galaxies which have been used for the accurate determination of the Hubble constant. These observational studies at present are being intensely continued (see e.g. Ref.[6],[7]), so that the most accurate study of the internal dynamics of the cluster is especially desirable, since each galaxy can participate a bulk flow motion, apart from the Hubble expansion.

To study the internal dynamics one has first to analyze the hierarchical structure of the cluster. By this first step we reveal the substructure of the Virgo cluster core within the $\pm 2$ arc degree field centered on M87 using the S-tree method. On this point the present study is the generalization of the previous study[8], where the Virgo cluster within central 1 arc degree field has been studied. For the second step - the reconstruction of the bulk regular flow of the subgroups, we use the method developed in Ref.[1] based on the data on the radial velocities of the members of the cluster. This method has been already applied to reveal the bulk motions within the Local Group of galaxies, and hence to obtain the apex and the 3D velocity vector of the motion of the Local Group with respect to the Cosmic Microwave Background frame[9].

For the Virgo cluster our analysis indicates the existence of 3 main subgroups of galaxies. Then, we estimate the transversal components of regular motions of the subgroups with respect to each other. Among the main results of this study we have to mention the unambiguous indication of the elongated nature of the Virgo cluster, and combined [8]with the rest kinematic parameters of the system we could even get constraints on the inclination angle and the degree of elongation.

The ongoing studies on better estimation of the distances of individual galaxies, particularly using the Cepheids, Tully-Fisher methods (e.g. [10],[11],[12], the method of Surface Brightness Fluctuations for Virgo's brightest galaxies[7], and other methods will enable from the results obtained here to determine more precisely the 3D bulk flow of each subgroups, and thus the mean motion of the Virgo cluster itself. Essential complementary information on the parameters of the clusters and groups of galaxies can be provided by the X-ray data[13].

## 2    Virgo cluster substructures

To study the substructures of the Virgo cluster core we used the data from the CfA redshift catalogue[14]. We dealt with the four arc degree central field within: N4316, RA= 12h 20m 12s and N4584, RA= 12h 35m 46.8s with the Virgo center coordinates RA=12h 27m 50.4s, DEC=12d 55m 55.2s. The sample includes 146 galaxies with available redshifts and V magnitudes.

For the description of S-tree method one can refer not only to the original works[15],[16], but also to the recent applications of this method to various clusters[1],[4], where it is discussed in details. Therefore here we only stress that this method is revealing the degree of mutual interaction of members of the system using self-consistently the information both on the 2D coordinates and redshifts of individual galaxies of the cluster as well as on their magnitudes.

The results of our study of the Virgo cluster core showed the existence of three main subgroups of galaxies containing 54, 17 and 37 galaxies, correspondingly. The lists of those galaxies along with their coordinates and redshifts are given in Tables 1, 2 and 3.

While comparing the results of this analysis with those of the 1 degree field[8] it is interesting to notice, that we see the strong correspondence with the 1 degree field membership results. This fact is remarkable since it shows the robust character of the results of subgrouping derived by S-tree method even when limited areas of the clusters are studied. Relative stability of the subgrouping at various scales follows also from physical considerations.

As seen from Table 4 below, though the mean redshifts of the subgroups are rather different, their redshift dispersions are quite close to each other. While the study of the regular motions of the subgroups will be done in next sections, already the remarkable difference of line-of-sight velocities is already indicating their possible significant radial bulk velocity with respect to each other.

## 3    Bulk flow reconstruction

The basic idea of the reconstruction procedure of the 3D motions within an N-body system from data on 1D line-of-sight velocities relies on the existence of correlation between the velocities of different members of an interacting system. From this point of view our scheme of 3D velocity reconstruction and the S-tree method have identical physical background. For the case of

Table 1: VIRGO subgroup I: 54 galaxies.

| Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ | Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ |
|------|-----|-----|--------|------|-----|-----|--------|
| N4316 | 280.722 | 70.963 | 1599 | N4371 | 279.666 | 73.369 | 1273 |
| N4374 | 278.183 | 74.475 | 1363 | N4377 | 275.310 | 76.206 | 1700 |
| N4379 | 273.742 | 76.967 | 1394 | N4380 | 281.900 | 71.820 | 1298 |
| N4390 | 281.819 | 72.270 | 1452 | I3328 | 282.325 | 71.897 | 1337 |
| I3331 | 280.458 | 73.575 | 1608 | A1223+1308 | 279.110 | 74.545 | 1594 |
| N4411 | 283.890 | 70.818 | 1616 | I3344 | 278.454 | 75.247 | 1702 |
| N4411 | 284.091 | 70.856 | 1605 | N4417 | 283.452 | 71.520 | 1178 |
| I3356 | 281.390 | 73.389 | 1433 | I3371 | 282.534 | 72.783 | 1260 |
| N4429 | 282.361 | 73.012 | 1463 | N4431 | 280.995 | 74.132 | 1243 |
| I3374 | 283.584 | 71.970 | 1202 | N4436 | 281.176 | 74.175 | 1454 |
| A1225+900 | 285.316 | 70.800 | 1442 | N4451 | 285.083 | 71.334 | 1195 |
| A1226+94 | 285.139 | 71.512 | 1374 | N4459 | 280.123 | 75.842 | 1540 |
| I3413 | 283.586 | 73.469 | 1735 | N4477 | 281.538 | 75.610 | 1680 |
| N4478 | 283.377 | 74.392 | 1698 | N4479 | 281.855 | 75.576 | 1183 |
| N4483 | 286.771 | 71.223 | 1215 | A1228+1219 | 284.137 | 74.159 | 1578 |
| N4486 | 283.761 | 74.489 | 1620 | N4497 | 285.185 | 73.806 | 1427 |
| I3457 | 284.374 | 74.818 | 1796 | N4503 | 286.081 | 73.413 | 1688 |
| I3468 | 287.006 | 72.516 | 1703 | I3468 | 285.632 | 74.122 | 1438 |
| A1230+926 | 288.084 | 71.497 | 1563 | N4515 | 280.529 | 78.319 | 1258 |
| N4516 | 283.225 | 76.739 | 1280 | I3487 | 288.443 | 71.748 | 1489 |
| N4519 | 289.177 | 71.049 | 1562 | I3499 | 287.655 | 73.337 | 1555 |
| N4528 | 287.628 | 73.675 | 1702 | I3510 | 287.986 | 73.446 | 1703 |
| I3518 | 289.268 | 72.045 | 1771 | I3520 | 285.897 | 75.816 | 1416 |
| N4540 | 283.618 | 77.801 | 1605 | A1232+927 | 289.947 | 71.644 | 1617 |
| N4551 | 288.150 | 74.681 | 1523 | N4564 | 289.541 | 73.920 | 1492 |
| I3583 | 288.272 | 75.709 | 1448 | I3602 | 291.863 | 72.660 | 1607 |

Table 2: VIRGO subgroup II: 17 galaxies.

| Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ | Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ |
|---|---|---|---|---|---|---|---|
| N4321 | 271.133 | 76.899 | 1884 | N4330 | 278.752 | 72.906 | 1885 |
| N4383 | 272.099 | 77.758 | 2015 | N4405 | 273.413 | 77.585 | 2063 |
| I3349 | 280.227 | 74.227 | 1801 | N4421 | 275.764 | 77.021 | 1925 |
| I3369 | 274.990 | 77.549 | 2048 | I3392 | 278.247 | 76.760 | 2001 |
| I796 | 276.345 | 78.126 | 1919 | N4474 | 280.797 | 76.005 | 1949 |
| N4486B | 283.392 | 74.563 | 1914 | I3457 | 284.374 | 74.818 | 1796 |
| I3470 | 286.240 | 73.508 | 1829 | I3501 | 285.437 | 75.594 | 1932 |
| I3586 | 289.098 | 75.005 | 1892 | N4578 | 291.682 | 72.125 | 2613 |
| N4579 | 290.374 | 74.361 | 1845 | | | | |

Table 3: VIRGO subgroup III: 37 galaxies.

| Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ | Name | $l$ | $b$ | $z_{\mathrm{CMB}}$ |
|---|---|---|---|---|---|---|---|
| I3239 | 278.239 | 73.229 | 979 | N4328 | 271.511 | 76.943 | 841 |
| N4387 | 278.828 | 74.465 | 913 | A1223+1513 | 275.513 | 76.439 | 830 |
| N4402 | 278.760 | 74.783 | 565 | I3363 | 280.318 | 74.349 | 1120 |
| N4424 | 283.865 | 71.389 | 767 | A1224+936 | 284.169 | 71.332 | 1047 |
| N4435 | 280.148 | 74.889 | 1101 | N4440 | 281.363 | 74.170 | 1068 |
| N4442 | 284.140 | 71.819 | 849 | N4445 | 284.625 | 71.482 | 635 |
| I3381 | 282.244 | 73.722 | 967 | I3393 | 281.313 | 74.817 | 794 |
| N4452 | 282.696 | 73.729 | 553 | N4458 | 281.078 | 75.151 | 1011 |
| I3412 | 284.978 | 72.086 | 1098 | A1226+1243 | 282.451 | 74.438 | 866 |
| N4469 | 286.087 | 70.905 | 833 | N4486A | 283.984 | 74.386 | 778 |
| N4491 | 284.808 | 73.636 | 826 | I3459 | 284.941 | 74.359 | 606 |
| I3466 | 285.451 | 74.028 | 1114 | N4506 | 283.826 | 75.578 | 1006 |
| I798 | 281.397 | 77.480 | 734 | N4523 | 283.106 | 77.362 | 582 |
| I3517 | 289.588 | 71.588 | 764 | I3522 | 284.036 | 77.476 | 980 |
| N4548 | 285.669 | 76.830 | 871 | I3540 | 287.350 | 75.326 | 1057 |
| N4550 | 288.087 | 74.635 | 706 | A1233+1239 | 288.036 | 74.793 | 608 |
| N4552 | 287.914 | 74.967 | 647 | A1233+1408 | 286.827 | 76.240 | 1078 |
| I3578 | 289.978 | 73.592 | 1021 | N4571 | 287.486 | 76.659 | 663 |
| A1235+1508 | 288.207 | 77.363 | 1055 | | | | |

stellar systems (large $N$) the problem of reconstruction of 3D velocity distribution of stars has been solved decades ago by Ambartsumian[17], without any a priori assumption on the form of the distribution function. The only natural assumption was its translation invariance, i.e. the phase space distribution of the system can be split as

$$dP = \Phi_S(x_1, x_2, x_3) dx_1 dx_2 dx_3 \times \Phi_K(v_1, v_2, v_3) dv_1 dv_2 dv_3, \qquad (1)$$

where $\Phi_S(x_1, x_2, x_3)$ is the 3D spatial distribution of objects and $\Phi_K(v_1, v_2, v_3)$ is the 3D velocity distribution function. The efficiency of such reconstruction procedure is however closely related to the number of objects considered. For the case of galaxy clusters (small $N$), it turns out that Ambartsumian's method cannot be applied directly. However some quantities of interest, for example the bulk flow of individual clusters, can be extracted from the data by assuming reasonable form of the velocity distribution function $\Phi_K$. Herein we assume that the velocities distribution of galaxies inside the cluster can be described by gaussian random isotropic components of velocity dispersion $\sigma_v$ plus the 3D mean peculiar velocity of the cluster, i.e.

$$\Phi_K(v_1, v_2, v_3) = g(v_1; V_1, \sigma_v) \, g(v_2; V_r, \sigma_v) \, g(v_3; V_3, \sigma_v), \qquad (2)$$

where $V_r$ is the radial component of the cluster bulk flow and $V_1$ and $V_3$ are its components perpendicular to the line-of-sight. Note however that this hypothesis does not imply any severe limitations in our problem. Moreover it is indeed justified by the fact that due to exponential instability of gravitating systems and their mixing properties, the correlations in physical parameters of particles have to split. This effect can be followed in numerical experiments.

Another limitation comes from the fact that galaxies inside a cluster take part in the Hubble flow, implying then that a fraction of the observed redshift is due to the spatial elongation of the cluster along the line-of-sight. A common practice is to assume that this effect is negligible, i.e. all the galaxies lie at the same distance. In Ref.[1] we used a more physical assumption allowing a 3D spatial extension of the cluster with gaussian isotropic distribution around the cluster center, i.e.

$$\Phi_S(x_1, x_2, x_3) = g(x_1; 0, \sigma_S) \, g(x_2; 0, \sigma_S) \, g(x_3; 0, \sigma_S), \qquad (3)$$

where $\sigma_S$ is the spatial dispersion of the cluster. Under this working assumption it is shown in Appendix A3 that the observed probability density

reads in terms of the redshift $z$ and the angular position with respect to the center of the cluster $\theta = (\theta_1, \theta_3)$ of the galaxies as

$$dP_{\text{obs}} = g(z - \langle z \rangle; V_1 \, \theta_1 + V_3 \, \theta_3, \sigma_{\text{obs}}) \times g(\theta_1; 0, \sigma_{\theta_1}) \, g(\theta_3; 0, \sigma_{\theta_3}) \, d\theta_3 d\theta_1 dz \quad (4)$$

where $\sigma_{\text{obs}}^2 = \sigma_v^2 + \sigma_S^2$, $\langle z \rangle$ is the mean redshift of the cluster and $\sigma_{\theta_1}$ and $\sigma_{\theta_3}$ are the angular sizes of the cluster in the plane perpendicular to the mean line-of-sight. It thus turns out from Eq. (4) that the tangential velocity of the cluster $V_{\text{tan}} = (V_1, V_3)$ can be evaluated by using standard statistical technique.

# 4  Application to Virgo subgroups

For each Virgo subgroup the system of coordinates $(x_1, x_2, x_3)$ is defined as follows. The $x_2$-axis is directed towards the mean angular position of subgroup galaxies $(\langle l \rangle, \langle b \rangle)$, the $x_1$-axis is parallel to the galactic plane and the $x_3$-axis is chosen in a way that the $(x_1, x_2, x_3)$ system forms a direct trihedron. The $x_1$, $x_2$ and $x_3$ coordinates then are deduced by transforming the cartesian galactic coordinates $(z \cos l \cos b, z \sin l \cos b, z \sin b)$ so that the two trihedrons coincide. Because the velocity dispersion $\sigma_v$ is small compared to the mean redshift $\langle z \rangle$ of the cluster, one has within the approximation of small angles $x_1 \approx \theta_1 \, \langle z \rangle$ and $x_3 \approx \theta_3 \, \langle z \rangle$ where $\theta_1$ and $\theta_3$ are the position angles along the $x_1$ and $x_3$-axis respectively, and $x_2 \approx z$.

The 3D data distribution of Virgo subgroup I is given in Figure 1, where the $x_2$ coordinate has been transformed to $x_2 = z - \langle z \rangle$ for convenience. Redshifts are expressed in km s$^{-1}$ in the CMB frame. The influence of peculiar velocities is clearly seen on $x_1 x_2$ and $x_2 x_3$ projections. Indeed the data distribution is much more elongated along the $x_2$-axis due to the presence of the velocity dispersion. The two left panels of the figure reveal a correlation between $\theta_1$ and $\theta_3$ (or equivalently between $x_1$ and $x_3$). Such a correlation which cannot be described by Eq. (4) clearly means that we have to modify our initial hypothesis, namely that the spatial distribution of the subgroup galaxies admits a central symmetry.

FIGURE 1.

Indeed the hypothesis of a central symmetry is stronger than it is necessary for the problem we are dealing with i.e. for the tangential bulk flow estimation. In Appendix A3 it is shown that the sufficient condition for

Table 4: Estimate of the parameters for the 3 Virgo subgroups. Units: $\langle l \rangle$ and $\langle b \rangle$ in degrees, $\sigma_{\theta_1}$, $\sigma_{\theta_3}$ and $\sigma'_{\theta_1}$ in radians and $\langle z \rangle$, $\sigma_{\text{obs}}$, $A_{21}$ and $A_{23}$ in km s$^{-1}$.

|  | Virgo I | Virgo II | Virgo III |
|---|---|---|---|
| $N_{\text{gal}}$ | 54 | 17 | 37 |
| $\langle z \rangle$ | 1497.91 | 1958.22 | 862.96 |
| $\langle l \rangle$ | 284.162 | 281.446 | 283.635 |
| $\langle b \rangle$ | 73.658 | 75.637 | 74.522 |
| $\sigma_{\theta_1}$ | 0.0182 | 0.0302 | 0.0176 |
| $\sigma_{\theta_3}$ | 0.0326 | 0.0338 | 0.0331 |
| $\sigma'_{\theta_1}$ | 0.0165 | 0.0183 | 0.0173 |
| $\rho_{13}$ | $-0.172$ | $-0.631$ | $-0.033$ |
| $A_{13}$ | $-0.23 \pm 0.07$ | $-0.71 \pm 0.14$ | $-0.10 \pm 0.09$ |
| $\sigma_{\text{obs}}$ | 167.22 | 171.26 | 178.95 |
| $A_{21}$ | $2485 \pm 1375$ | $2684 \pm 2264$ | $-489 \pm 1703$ |
| $A_{23}$ | $938 \pm 767$ | $476 \pm 2023$ | $-109 \pm 904$ |

evaluating $V_{\text{tan}}$ is the absence of correlation between the spatial distribution of galaxies along the line-of-sight and perpendicular to it, i.e. $a_{21}$ and $a_{23}$ are vanishing in Eq. (46). In this case, the observed distribution function takes the form

$$
\begin{aligned}
dP_{\text{obs}} \quad &= g(z - \langle z \rangle; A_{21}\,\theta_1 + A_{23}\,\theta_3, \sigma_{\text{obs}}) \\
&\times g(\theta_1; A_{13}\,\theta_3, \sigma'_{\theta_1})\, g(\theta_3; 0, \sigma_{\theta_3})\, d\theta_3 d\theta_1 dz
\end{aligned}
\tag{5}
$$

where the tangential velocities are $V_1 \equiv A_{21}$ and $V_2 \equiv A_{23}$. Here the parameter $A_{13}$ allows us to include into consideration the correlation between $\theta_1$ and $\theta_3$, as required by the observational data.

The estimates of tangential bulk flows of the three Virgo subgroups are given in Table 4 using the formulae derived in Appendix A3. Due to the relatively small number of galaxies in the subgroups the error boxes are large, but nevertheless are informative. Note, that for subgroups I and II the bulk flow peculiar tangential velocities are anomaly high. These values are significant i.e. are not due to the sampling errors.

# 5    The spatial structure of Virgo subgroups

The statistically significant correlation between $\theta_1$ and $\theta_3$ for Virgo subgroups I and II advocates against the hypothesis of central symmetry for the spatial distribution of their galaxies. This means that these systems represent spatially elongated configurations. Their elongation can be described by the following ellipsoidal 3D spatial distribution function

$$\Phi_{\mathrm{S}}(y_1, y_2, y_3) = g(y_1; 0, \sigma_1)\, g(y_2; 0, \sigma_2)\, g(y_3; 0, \sigma_3) \tag{6}$$

where at least one of the dispersions, say $\sigma_1$, differs from the two others. In this case, the orientation of the proper system of coordinates $(y_1, y_2, y_3)$ of the ellipsoidal distribution has to be defined with respect to the frame of analysis, namely $(x_1, x_2, x_3)$. This is done by introducing 3 rotation angles $\alpha$, $\beta$ and $\gamma$, where the angles $\alpha$ and $\beta$ are respectively the longitude and the latitude of the $y_1$-axis in the $(x_1, x_2, x_3)$ frame and $\gamma$ is the angle between the intersection of the planes $x_1 x_2$ and $y_2 y_3$ and the $y_2$-axis. As it is shown in Appendix A2 the spatial probability density in the coordinate frame $(x_1, x_2, x_3)$ has to form

$$dP_{\mathrm{S}} = g(x_3; 0, \sigma_{\mathrm{I}})\, g(x_1; a_{13}\, x_3, \sigma_{\mathrm{II}}) \times g(x_2; a_{21}\, x_1 + a_{23}\, x_3, \sigma'_{\mathrm{III}})\, dx_1 x_2 x_3 \tag{7}$$

where the parameters $a_{13}$, $a_{21}$, $a_{23}$, $\sigma_{\mathrm{I}}$, $\sigma_{\mathrm{II}}$ and $\sigma'_{\mathrm{III}}$ are linked to the proper spatial dispersions $\sigma_1$, $\sigma_2$ and $\sigma_3$ and the orientation angles $\alpha$, $\beta$ and $\gamma$ by the formulae given in that appendix. The case $a_{21} = 0$ and $a_{23} = 0$ arises if the ellipsoid is symmetrical with respect to the plane perpendicular to the line-of-sight ($\alpha = 0$ and $\gamma = 0$).

A it is shown in the appendix A3, the observed distribution function has the same form as in Eq. (5), but then the parameters $A_{21}$ and $A_{23}$ are

$$A_{21} = V_1 + a_{21}\, H_0\, D \quad ; \quad A_{23} = V_3 + a_{23}\, H_0\, D \tag{8}$$

where $H_0$ is the Hubble's constant and $D$ is the distance of the considered subgroup. These expressions show us that in general it is not possible to disentangle between the presence of tangential subgroup bulk flow and the effect of inclination along the line-of-sight of the ellipsoidal distribution of subgroup galaxies. In particular, it is very likely that the large values of $A_{21}$ and $A_{23}$ obtained for the Virgo I and II subgroups are partly due to this inclination effect.

Finally, let us consider the spatial structure of Virgo su bgroups I and II based on the results given in Table 4. The initial model contains 10 free

parameters, namely the 3 bulk flow components $V_r$, $V_1$ and $V_3$, the velocity dispersion $\sigma_v$, the 3 proper spatial dispersions $\sigma_1$, $\sigma_2$ and $\sigma_3$ and the 3 orientation angles $\alpha$, $\beta$ and $\gamma$. Our approach enables us to estimate the 6 independent parameters $A_{13}$, $A_{23}$, $A_{21}$, $\sigma'_{\theta_1}$, $\sigma_{\theta_3}$ and $\sigma_{\text{obs}}$. Assuming that the subgroup is at rest in the CMB frame, i.e. $V_r = 0$, $V_1 = 0$ and $V_3 = 0$, one can deduce for a given value of the velocity dispersion $\sigma_v$ the characteristics of the spatial structure of the subgroup.

We have performed this experiment for Virgo subgroups I and II based on the data in Table 4. Figures 2 and 3 show respectively the variation of parameters $\sigma_1$, $\sigma_2$, $\sigma_3$, $\alpha$, $\beta$ and $\gamma$ as a function of $\sigma'_{\text{III}} = \sqrt{\sigma_{\text{obs}}^2 - \sigma_v^2}$ for the subgroups I and II. Since the problem does not admit a general analytical solution, these curves have been obtained numerically.

The results show that both subgroups reveal an elongated structure with significant difference between $\sigma_1$, $\sigma_2$ and $\sigma_3$, whatever is the contribution of the true velocity dispersion $\sigma_v$ in the observed redshift dispersion $\sigma_{\text{obs}}$.

FIGURES 2 and 3.

# 6   Conclusion

Thus the technique we had developed for 3D velocity reconstruction of galaxy configurations[9] notwithstanding to the inevitable error boxes, can be successfully applied to the Virgo cluster subgroups.

By a first step we have estimated the tangential bulk flow velocity of the 3 subgroups under the assumption that the observed redshift dispersion is not due to their 3D spatial structure. For two of the subgroups, namely Virgo I and Virgo II, these values were found statistically significant but anormaly high (2656 km s$^{-1}$ and 2726 km s$^{-1}$ respectively). This fact indicates the validity of our main working hypothesis, i.e. that the clustered galaxies exhibit an elongated spatial distribution along the line-of-sight.

We have shown that the knowledge of galaxies redshifts alone still does not permit to disentangle between the presence of tangential bulk flow and the effect of inclination of an elongated 3D spatial structure. Nevertheless, while both effects are certainly contributing to our derived statistics, we can assert unambiguously that both subgroups are spatially elongated, with the specific characteristics of their 3D structures depending on the kinematic properties of the subgroups.

## Acknowledgements

## A    Useful formulae

The probability density of a random variable $x$ given as a gaussian distribution with a mean $x_0$ and dispersion $\sigma_x$

$$dP_{\mathrm{G}} = g(x; x_0, \sigma_x)\, dx = \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp\left(-\frac{(x - x_0)^2}{2\sigma_x^2}\right) dx \qquad (9)$$

fulfills the properties

$$g(x + \lambda; x_0, \sigma_x)\, dx = g(x; x_0 - \lambda, \sigma_x)\, dx \qquad (10)$$

$$g(\lambda\, x; \lambda\, x_0, |\lambda|\, \sigma_x)\, d(|\lambda|\, x) = g(x; x_0, \sigma_x)\, dx, \qquad (11)$$

where $\lambda$ is a real number. The following relation also holds

$$g(x; x_1, \sigma_1)\, g(x; x_2, \sigma_2) = g(x; x_0, \sigma_0)\, g(x_1; x_2, \sigma), \qquad (12)$$

with $x_0$, $\sigma_0$ and $\sigma$ defined as

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \qquad ; \qquad \sigma_0^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}. \qquad (13)$$

$$x_0 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\, x_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\, x_2. \qquad (14)$$

We describe an ellipsoidal 2-dimensional distribution in the system of coordinates $(y_1, y_2)$ by the probability density

$$dP_{\mathrm{ell}} = g(y_1; 0, \sigma_1)\, dy_1 \times g(y_2; 0, \sigma_2)\, dy_2 \qquad (15)$$

or using the properties (10), (11) and (12) in the system of coordinates $(x_1, x_2)$

$$dP_{\mathrm{ell}} = g(x_1; A\, x_2, \sigma_1')\, g(x_2; 0, \sigma_2')\, dx_1 dx_2. \qquad (16)$$

with $\sigma_2'$, $\sigma_1'$ and $A$ defined as follows

$$\sigma_2'^{\,2} = \sigma_2^2 \cos^2\alpha + \sigma_1^2 \sin^2\alpha \qquad ; \qquad \sigma_1' = \frac{\sigma_1 \sigma_2}{\sigma_2'} \qquad (17)$$

$$A = \sin\alpha\cos\alpha\left(\frac{\sigma_1^2 - \sigma_2^2}{\sigma_2'^2}\right). \tag{18}$$

Here the angle $\alpha$ characterizes the rotation $R_\alpha$ transforming $(x_1, x_2)$ into $(y_1, y_2)$ coordinates, i.e.

$$R_\alpha \; : \; \begin{cases} y_1 = \cos\alpha \; x_1 + \sin\alpha \; x_2 \\ y_2 = -\sin\alpha \; x_1 + \cos\alpha \; x_2. \end{cases} \tag{19}$$

# B   The spatial distribution

The 3-dimensional spatial distribution of considered galaxies can be expressed in the system of coordinates $(y_1, y_2, y_3)$ as

$$dP_{\mathrm{S}} = g(y_1; 0, \sigma_1)\, dy_1 \times g(y_2; 0, \sigma_2)\, dy_2 \times g(y_3; 0, \sigma_3)\, dy_3. \tag{20}$$

The spatial orientation of this trihedron is herein characterized by 3 rotation angles $\alpha$, $\beta$ and $\gamma$ with respect to the frame of analysis $(x_1, x_2, x_3)$. The angles $\alpha$ and $\beta$ are respectively the longitude and the latitude of the $y_1$-axis such that coordinate transformations read as

$$R_\alpha \; : \; \begin{cases} y_1'' = \cos\alpha \; x_1 + \sin\alpha \; x_2 \\ y_2'' = -\sin\alpha \; x_1 + \cos\alpha \; x_2 \\ y_3'' = x_3, \end{cases} \tag{21}$$

where $(y_1'', y_2'', y_3'')$ is the new system of coordinates after a rotation $R_\alpha$ of angle $\alpha$ around the $x_3$-axis,

$$R_\beta \; : \; \begin{cases} y_1' = \cos\beta \; y_1'' + \sin\beta \; y_3'' \\ y_2' = y_2'' \\ y_3' = -\sin\beta \; y_1'' + \cos\beta \; y_3'' \end{cases} \tag{22}$$

Here $(y_1', y_2', y_3')$ is the system of coordinates after rotating on an angle $\beta$ with repect the $y_2''$ axis, and the angle $\gamma$ is defined by a rotation $R_\gamma$ around $y_1'$ axis so that $y_2'$ and $y_2$ axes coincide, i.e.

$$R_\gamma \; : \; \begin{cases} y_1 = y_1' \\ y_2 = \cos\gamma \; y_2' + \sin\gamma \; y_3' \\ y_3 = -\sin\gamma \; y_2' + \cos\gamma \; y_3'. \end{cases} \tag{23}$$

Our aim is to express the density probability of Eq. (20) in terms of the coordinates of analysis, namely $(x_1, x_2, x_3)$. Using (16) and in view of (23), one has

$$
\begin{aligned}
dF_1 \quad &= g(y_2; 0, \sigma_2)\, dy_2\, g(y_3; 0, \sigma_3)\, dy_3 \\[2mm]
&= g(y_2'; A\, y_3', \sigma_2')\, g(y_3'; 0, \sigma_3')\, dy_2' dy_3',
\end{aligned}
\tag{24}
$$

with $\sigma_3'$, $\sigma_2'$ and $A$ defined as

$$
{\sigma_3'}^{\,2} = \sigma_3^2 \cos^2 \gamma + \sigma_2^2 \sin^2 \gamma \quad ; \quad \sigma_2' = \frac{\sigma_2 \sigma_3}{\sigma_3'},
\tag{25}
$$

$$
A = \sin \gamma \cos \gamma \left( \frac{\sigma_2^2 - \sigma_3^2}{{\sigma_3'}^{\,2}} \right).
\tag{26}
$$

The relation (16) and definitions (22,21,23) yield

$$
\begin{aligned}
dF_2 \quad &= g(y_1; 0, \sigma_1)\, dy_1\, g(y_3'; 0, \sigma_3')\, dy_3' \\[2mm]
&= g(y_1''; B\, x_3, \sigma_1'')\, g(x_3; 0, \sigma_3'')\, dy_1'' dx_3,
\end{aligned}
\tag{27}
$$

with $\sigma_3''$, $\sigma_1''$ and $B$ defined as

$$
{\sigma_3''}^{2} = {\sigma_3'}^{\,2} \cos^2 \beta + \sigma_1^2 \sin^2 \beta \quad ; \quad \sigma_1'' = \frac{\sigma_1 \sigma_3'}{\sigma_3''}
\tag{28}
$$

$$
B = \sin \beta \cos \beta \left( \frac{\sigma_1^2 - {\sigma_3'}^{\,2}}{{\sigma_3''}^{2}} \right).
\tag{29}
$$

Using the relations (10) and (11), the definitions (21,22) and defining $b_1$, $b_2$, $b_3$, $b_4$, $b_5$ and $b_6$ as

$$
\begin{aligned}
b_1 &= \sin \alpha & b_2 &= \cos \alpha + A \sin \alpha \sin \beta \\
b_5 &= -\cos \alpha & b_3 &= \sin \alpha - A \cos \alpha \sin \beta \\
b_4 &= A \cos \beta & b_6 &= B.
\end{aligned}
\tag{30}
$$

we obtain for the following probability density

$$
\begin{aligned}
dF_3 \quad &= g(y_2'; A\, y_3', \sigma_2')\, g(y_1''; B\, x_3, \sigma_1'')\, dy_2' dy_1'' \\[2mm]
&= g(b_2\, x_2; b_3\, x_1 + b_4\, x_3, \sigma_2')\, d|b_2 x_2| \\
&\quad \times g(b_1\, x_2; b_5\, x_1 + b_6\, x_3, \sigma_1'')\, d|b_5 x_1| \\[2mm]
&= g(b_1 b_2\, x_2; b_1 b_3\, x_1 + b_1 b_4\, x_3, |b_1| \sigma_2')\, d|b_1 b_2 x_2| \\
&\quad \times g(b_2 b_1\, x_2; b_2 b_5\, x_1 + b_2 b_6\, x_3, |b_2| \sigma_1'')\, d|b_2 b_5 x_1|.
\end{aligned}
$$

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Using (12) and since $b_1 b_3 - b_2 b_5 = 1$ the spatial distribution of Eq. (20) can be rewritten in terms of the coordinates $(x_1, x_2, x_3)$

$$
\begin{aligned}
dP_{\text{S}} \quad &= g(x_3; 0, \sigma_{\text{I}})\, g(x_1; a_{13}\, x_3, \sigma_{\text{II}}) \\[2mm]
&\times g(x_2; a_{21}\, x_1 + a_{23}\, x_3, \sigma'_{\text{III}})\, dx_1 x_2 x_3,
\end{aligned}
\tag{31}
$$

where the parameters $a_{13}$, $a_{21}$, $a_{23}$, $\sigma_{\text{I}}$, $\sigma_{\text{II}}$ and $\sigma'_{\text{III}}$ are obtained from Eqs. (30,25,28,26,29)

$$
\sigma_{\text{II}}^2 = b_1^2\, \sigma_2'^2 + b_2^2\, \sigma_1''^2 \quad ; \quad a_{13} = b_2\, b_6 - b_1\, b_4
\tag{32}
$$

$$
\sigma_{\text{I}} = \sigma_3'' \quad ; \quad a_{21} = \frac{1}{\sigma_{\text{II}}^2}\, (b_3 b_2 \sigma_1''^2 - b_5\, b_1 \sigma_2'^2)
\tag{33}
$$

$$
\sigma'_{\text{III}} = \frac{\sigma_1'' \sigma_2'}{\sigma_{\text{II}}} \quad ; \quad a_{23} = \frac{1}{\sigma_{\text{II}}^2}\, (b_4 b_2 \sigma_1''^2 - b_6\, b_1 \sigma_2'^2)
\tag{34}
$$

The problem of extracting the values of parameters $(\alpha, \beta, \gamma, \sigma_1, \sigma_2, \sigma_3)$ from a given set of observed parameters $(a_{13}, a_{21}, a_{23}, \sigma_{\text{I}}, \sigma_{\text{II}}, \sigma_{\text{III}})$ is not straightforward in general. The special case of a 2-dimensional spatial distribution deserves mentioning however. If one of the dispersions in Eq. (20) vanishes (e.g. $\sigma_1 = 0$), the previous calculations lead to

$$
\sigma_1'' = 0 \quad ; \quad \sigma_2'^2 = \frac{\sigma_{\text{II}}^2}{\sin^2 \alpha} \quad ; \quad \sigma_3'^2 = \frac{\sigma_{\text{I}}^2}{\cos^2 \beta}
$$

$$
A = -\frac{\cos \beta}{\sin \alpha}\, (a_{13} + \cos \alpha \, \tan \beta) \quad ; \quad B = -\tan \beta,
$$

which implies

$$
\tan \alpha = -\frac{1}{a_{21}} \quad ; \quad \tan \beta = -a_{23} \sin \alpha
\tag{35}
$$

and finally yields

$$
\sinh(\ln(\tan \gamma)) = -\frac{1}{A}\left( A^2 - 1 + \frac{\sigma_{\text{II}}^2}{\sigma_{\text{I}}^2}\, \frac{\cos^2 \beta}{\sin^2 \alpha} \right)
\tag{36}
$$

$$
\sigma_2^2 = \left( 1 + \frac{A}{\tan \gamma} \right) \frac{\sigma_{\text{I}}^2}{\cos^2 \beta}
\tag{37}
$$

$$
\sigma_3^2 = (1 - A \tan \gamma)\, \frac{\sigma_{\text{I}}^2}{\cos^2 \beta}.
\tag{38}
$$

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# C  Distribution in the phase space

The phase space distribution of the considered dynamical system can be split as follows

$$dP_{\mathrm{PS}} = dP_{\mathrm{K}} \times dP_{\mathrm{S}}, \qquad (39)$$

where $dP_{\mathrm{S}}$ is the 3D spatial distribution of Eq. (20) and the probability density $dP_{\mathrm{K}}$ describes the distribution of peculiar velocities $\mathbf{v} = (v_1, v_2, v_3)$. We chose the system of coordinates such that the $x_2$-axis points toward the mean angular position of the cluster and such that the $x_1$-axis is parallel to the galactic plane; $v_1$, $v_2$ and $v_3$ are thus components of the galaxies peculiar velocity respectively along $x_1$, $x_2$ and $x_3$ axes. In this frame the 3D velocity distribution is given by the 3D mean peculiar velocity of the cluster plus random isotropic components of velocity dispersion $\sigma_v$, i.e.

$$dP_{\mathrm{K}} = g(v_1; V_1, \sigma_v)dv_1 \, g(v_2; V_r, \sigma_v)dv_2 \, g(v_3; V_3, \sigma_v)dv_3, \qquad (40)$$

where $V_r$ is the radial component of the bulk flow and $V_1$ and $V_3$ are its components perpendicular to the line-of-sight. The distance of the cluster is $D$ and the angular positions with respect to the center of the cluster are $\theta_1$ and $\theta_3$. In the approximation of small angles, $\theta_1$ and $\theta_3$ are

$$\theta_1 \approx \frac{x_1}{D} \quad ; \quad \theta_3 \approx \frac{x_3}{D}. \qquad (41)$$

Then the redshift $z$ of a galaxy expressed in km s$^{-1}$ units is

$$\begin{aligned} z \ &= H_0 \, D + \mathbf{v}.\mathbf{u} \\ &\approx H_0 \, D + v_2 + H_0 \, x_2 + \theta_1 \, v_1 + \theta_3 \, v_3, \end{aligned} \qquad (42)$$

where $H_0$ is the Hubble's constant and $\mathbf{u}$ is the line-of-sight velocity.

Integrating the probability density $dP_{\mathrm{K}}$ over the two unobserved tangential components of the velocity $v_1$ and $v_3$ we have

$$dP_z = g(z; H_0 \, D + V_r + H_0 \, x_2 + \theta_1 \, v_1 + \theta_3 \, v_3, \sigma_v) \, dz \qquad (43)$$

Where (10), (11) and (12) have been used. It follows from this result and from the expression Eq. (31) for $dP_{\mathrm{S}}$ the integral $dP_{\mathrm{PS}}$ in Eq. (39) over the unobserved variable $x_2$ yields

$$\begin{aligned} dP_{\mathrm{obs}} \ &= g(z; H_0 \, D + V_r + A_{21} \, \theta_1 + A_{23} \, \theta_3, \sigma_{\mathrm{obs}}) \\ &\times g(\theta_1; A_{13} \, \theta_3, \sigma'_{\theta_1}) \, g(\theta_3; 0, \sigma_{\theta_3}) \, d\theta_3 d\theta_1 dz, \end{aligned} \qquad (44)$$

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where the parameters $A_{13}$, $A_{21}$, $A_{23}$, $\sigma_{\rm obs}$, $\sigma_{\theta_3}$ and $\sigma'_{\theta_1}$ are related to $a_{13}$, $a_{21}$, $a_{23}$, $\sigma_{\rm I}$, $\sigma_{\rm II}$ and $\sigma'_{\rm III}$ from Eqs. (32,33,34). This characterize the spatial structure of the cluster, and velocity dispersion $\sigma_v$ and cluster's tangential velocities $V_1$ and $V_3$ through the following formulae

$$\sigma_{\rm obs}^2 = \sigma_v{}^2 + \sigma'_{\rm III}{}^2 \quad ; \quad A_{13} = a_{13} \tag{45}$$

$$A_{21} = V_1 + a_{21}\, H_0\, D \quad ; \quad A_{23} = V_3 + a_{23}\, H_0\, D \tag{46}$$

$$\sigma'_{\theta_1} = \frac{\sigma_{\rm II}}{H_0\, D} \quad ; \quad \sigma_{\theta_3} = \frac{\sigma_{\rm I}}{H_0\, D}. \tag{47}$$

So far as the angles $\theta_1$ and $\theta_3$ have been defined in a way that their averages over the sample vanish (i.e. $\langle\theta_1\rangle = 0$ and $\langle\theta_3\rangle = 0$), the observed probability density $dP_{\rm obs}$ of Eq. (44) can be rewritten as

$$dP_{\rm obs} \quad = g(Z; A_{21}\,\theta_1 + A_{23}\,\theta_3, \sigma_{\rm obs}),$$

$$\times g(\theta_1; A_{13}\,\theta_3, \sigma'_{\theta_1})\, g(\theta_3; 0, \sigma_{\theta_3})\, d\theta_3 d\theta_1 dZ, \tag{48}$$

where the observable $Z$ is defined as

$$Z = z - \langle z\rangle = z - (H_0\, D + V_r). \tag{49}$$

From Eq. (48) it turns out that the parameters $A_{13}$, $A_{21}$ and $A_{23}$ can be obtained using a standard multiple regression technique, i.e.

$$A_{13} = \frac{{\rm Cov}(\theta_1, \theta_3)}{{\rm Cov}(\theta_3, \theta_3)}, \tag{50}$$

$$A_{23} = \frac{{\rm Cov}(\theta_1, \theta_3){\rm Cov}(\theta_1, Z) - {\rm Cov}(\theta_1, \theta_1){\rm Cov}(\theta_3, Z)}{{\rm Cov}(\theta_1, \theta_3){\rm Cov}(\theta_1, \theta_3) - {\rm Cov}(\theta_1, \theta_1){\rm Cov}(\theta_3, \theta_3),} \tag{51}$$

$$A_{21} = \frac{{\rm Cov}(\theta_3, \theta_3){\rm Cov}(\theta_1, Z) - {\rm Cov}(\theta_1, \theta_3){\rm Cov}(\theta_3, Z)}{{\rm Cov}(\theta_1, \theta_1){\rm Cov}(\theta_3, \theta_3) - {\rm Cov}(\theta_1, \theta_3){\rm Cov}(\theta_1, \theta_3),} \tag{52}$$

where ${\rm Cov}(x, y) = \langle(x - \langle x\rangle)(y - \langle y\rangle)\rangle$ is the covariance of random variables $x$ and $y$. Estimates of the dispersions $\sigma_{\rm obs}$, $\sigma_{\theta_3}$ and $\sigma'_{\theta_1}$ are obtained using

$$\sigma_{\theta_3}{}^2 = {\rm Cov}(\theta_3, \theta_3), \tag{53}$$

$$\sigma'_{\theta_1}{}^2 = \sigma_{\theta_1}{}^2 - 2\, A_{13}\, {\rm Cov}(\theta_1, \theta_3) + A_{13}^2\, \sigma_{\theta_3}{}^2, \tag{54}$$

$$\sigma_{\rm obs}{}^2 = \sigma_Z{}^2 - 2\, A_{21}\, {\rm Cov}(\theta_1, Z) - 2\, A_{23}\, {\rm Cov}(\theta_3, Z), \tag{55}$$

$$+A_{21}^2\,\sigma_{\theta_1}{}^2 + A_{23}^2\,\sigma_{\theta_3}{}^2 - 2\,A_{21}\,A_{23}\,\mathrm{Cov}(\theta_3,\theta_1)$$

, where $\sigma_{\theta_1}$ and $\sigma_Z$ are defined as

$$\sigma_{\theta_1}{}^2 = \mathrm{Cov}(\theta_1,\theta_1) \quad ; \quad \sigma_Z{}^2 = \mathrm{Cov}(Z,Z). \tag{56}$$

Finally, the standard deviations i.e. accuracies of the estimators of parameters $A_{13}$, $A_{21}$ and $A_{23}$ given Eqs. (50,51,52) can be represented as a function of the number $N$ of galaxies in the cluster

$$\Delta A_{13} = \frac{1}{\sqrt{N}}\,\frac{\sigma'_{\theta_1}}{\sigma_{\theta_3}}, \tag{57}$$

$$\Delta A_{23} = \frac{1}{\sqrt{N}}\,\frac{1}{\sqrt{1-\rho_{31}^2}}\,\frac{\sigma_{\mathrm{obs}}}{\sigma_{\theta_3}}, \tag{58}$$

$$\Delta A_{21} = \frac{1}{\sqrt{N}}\,\frac{1}{\sqrt{1-\rho_{31}^2}}\,\frac{\sigma_{\mathrm{obs}}}{\sigma_{\theta_1}}, \tag{59}$$

where $\rho_{31}$ is the correlation coefficient between variables $\theta_1$ and $\theta_3$, i.e.

$$\rho_{31}^2 = \frac{\mathrm{Cov}(\theta_1,\theta_3)\,\mathrm{Cov}(\theta_1,\theta_3)}{.}\mathrm{Cov}(\theta_1,\theta_1)\,\mathrm{Cov}(\theta_3,\theta_3) = A_{13}^2\,\frac{\sigma_{\theta_3}^2}{\sigma_{\theta_1}^2} \tag{60}$$

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Figure captions.

Figure 1.
Virgo subgroup I: 54 galaxies. The top left panel visualizes the 3D information contained in the data. The $\theta_1\theta_3$ plane is perpendicular to the mean line-of-sight ($\langle l \rangle = 284.162, \langle b \rangle = 73.658$). The redshift depth is proportional to the radius of the circles, backside galaxies are shadow, frontside plain. The 3 other plots are the projection of this 3D distribution on plane $x_1x_3$, $x_2x_3$ and $x_1x_2$ respectively where $x_2 = z - \langle z \rangle$.

Figure 2.
Virgo subgroup I: Variation in function of $\sigma'_{\mathrm{III}}$ of the parameters characterizing the ellipsoidal spatial structure ($\sigma_1$, $\sigma_2, \sigma_3$) and its orientation ($\alpha, \beta, \gamma$). The dispersion $\sigma'_{\mathrm{III}}$ reads in terms of $\sigma_{\mathrm{obs}} = 167.22$ km s$^{-1}$ and the velocity dispersion $\sigma_v$ as $\sigma'_{\mathrm{III}} = \sqrt{\sigma_{\mathrm{obs}}^2 - \sigma_v^2}$.

Figure 3.
Same caption as fig. 2 but for Virgo subgroup II: The observed dispersion is $\sigma_{\mathrm{obs}} = 171.26$ km s$^{-1}$.

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